

Notes on Ghost Dark Energy

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ABSTRACT: We study a phenomenological dark energy model which is rooted in the Veneziano ghost of QCD. In this dark energy model, the energy density of dark energy is proportional to Hubble parameter and the proportional coefficient is of the order Λ_{QCD}^3 , where Λ_{QCD} is the mass scale of QCD. The universe has a de Sitter phase at late time and begins to accelerate at redshift around $z_{acc} \sim 0.6$. We also fit this model and give the constraints on model parameters, with current observational data including SnIa, BAO, CMB, BBN and Hubble parameter data. We find that the squared sound speed of the dark energy is negative, which may cause an instability. We also study the cosmological evolution of the dark energy with interaction with cold dark matter.

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1. Introduction

It has been more than ten years since our universe was found to be in accelerating expansion [1]. A new energy component of the universe, called dark energy (DE), is needed to explain this acceleration. The simplest model of DE is the cosmological constant, which is a key ingredient in the Λ CDM model. Although the Λ CDM model is consistent very well with all observational data, it faces with the fine tuning problem [2]. Plenty of other DE models have also been proposed [3, 4, 5, 6, 7, 40, 41], but almost all of them explain the acceleration expansion either by introducing new degree(s) of freedom or by modifying gravity.

Recently a new DE model, so-called Veneziano ghost DE, has been proposed [8, 9]. The key ingredient of this new model is that the Veneziano ghost, which is unphysical in the usual Minkowski spacetime QFT, exhibits important physical effects in dynamical spacetime or spacetime with non-trivial topology. Veneziano ghost is supposed to exist for solving the $U(1)$ problem in low-energy effective theory of QCD [10, 11, 12, 13, 14]. The ghost has no contribution to the vacuum energy density in Minkowski spacetime, but in curved spacetime it gives rise to a small vacuum energy density proportional to $\Lambda_{QCD}^3 H$, where Λ_{QCD} is QCD mass scale and H is Hubble parameter. This small vacuum energy density expects to play some role in the evolution of the universe. Because this model is totally embedded in standard model and general relativity, one needs not to introduce any new parameter, new degree of freedom or to modify gravity. With $\Lambda_{QCD} \sim 100 MeV$ and $H \sim 10^{-33} eV$, $\Lambda_{QCD}^3 H$ gives the right order of observed DE energy density. This numerical

coincidence is impressive and also means that this model gets rid of fine tuning problem [8, 9]. Actually, several authors have already suggested DE model with energy density proportional to $\Lambda_{QCD}^3 H$ in different physical contexts [15, 16, 17, 18, 19, 20, 21, 39, 40] such as QCD trace anomaly, gluon condensate of quantum chromodynamics, modifying gravity and so on.

In this work, we investigate the phenomenological model with energy density of DE ρ_{DE} proportional to Hubble parameter H . We study the cosmological evolution of the DE model with/without interaction between the DE and dark matter. We analytically and numerically compute some quantities such as scale factor a , ρ_{DE} , squared adiabatic speed of sound c_s^2 and so on. Also we fit this model with current observational data and give constraints on the model parameters.

The note is organized as follows. In section 2 we study the dynamical evolution of the DE model. In section 3, we fit this model with current observational data and discuss the fitting results. The data used are Union II SnIa sample [22], BAO data from SDSS DR7 [23], CMB data (R, l_a, z_*) from WMAP7 [24], 12 Hubble evolution data [25, 26] and Big Bang Nucleosynthesis (BBN) [27, 28]. In section 4, we calculate c_s^2 and find it is always negative. Negative c_s^2 may give rise to the instability problem. On the one hand, to try to avoid this problem, on the other hand, to further study the ghost DE model, we introduce the interaction between DE and cold dark matter (CDM), and study the dynamical evolution in this case. We summary our work and give some discussions in section 5.

2. Dynamics of ghost dark energy

To study the dynamics of the DE model, we consider a flat FRW universe with only two energy components, CDM and DE and neglect radiation and baryon temporarily in this section. We will include the radiation and baryon when fit the model with observational data in section 4.

In this ghost DE model, the energy density of DE is given by $\rho_{DE} = \alpha H$, where α is a constant with dimension $[energy]^3$, and roughly of order of Λ_{QCD}^3 where $\Lambda_{QCD} \sim 100\text{MeV}$ is QCD mass scale. Arming with this DE density, the Friedman equation reads

$$H^2 = \frac{8\pi G}{3} (\alpha H + \rho_m), \quad (2.1)$$

where ρ_m is energy density of CDM, whose continuity equation gives

$$\dot{\rho}_m + 3H\rho_m = 0 \implies \rho_m = \rho_{m0}a^{-3}. \quad (2.2)$$

We have set $a_0 = 1$ and the subscript 0 stands for the present value of some quantity. From (2.1) and (2.2), we can obtain the Raychaudhuri equation

$$\dot{H} + H^2 = -\frac{4\pi G}{3} \left[-\rho_{DE} \left(\frac{\dot{\rho}_{DE}}{H\rho_{DE}} + 2 \right) + \rho_m \right]. \quad (2.3)$$

Solve the Friedman equation, we have

$$H_{\pm} = \frac{4\pi G}{3}\alpha \pm \sqrt{\left(\frac{4\pi G}{3}\alpha\right)^2 + \frac{8\pi G}{3}\rho_{m0}a^{-3}}. \quad (2.4)$$

There are two branches, H_+ represents an expansion solution, while H_- a contraction one. We neglect the latter since it goes against the observation, and for simplicity, write H_+ as H in what follows.

To facilitate our discussion, we define a characteristic scale factor a_*

$$a_* \equiv \left(\frac{12\rho_{m0}}{8\pi G\alpha^2}\right)^{\frac{1}{3}} = \left(4\frac{\Omega_{m0}}{\Omega_{DE0}^2}\right)^{\frac{1}{3}} = 10^2 \left(36\Omega_{m0}\frac{\text{MeV}^6}{\alpha^2}\right)^{\frac{1}{3}},$$

where we have taken $H_0 = 10^{-33}eV$, Ω_{m0} and Ω_{DE0} are the dimensionless energy density of CDM and DE, respectively. One can see shortly that actually a_* is the transition point when the universe transits from the dust phase to a de Sitter phase. If we assume $\Omega_{m0} = \frac{1}{4}$ and $\Omega_{DE0} = \frac{3}{4}$, by definition, we get roughly $a_* \sim 1$, which means that the transition occurs just at present. Therefore, we will take $a_* = 1$ throughout this section. And especially $\alpha \sim (10\text{MeV})^3$ if $a_* \sim 1$.

At early epoch $a \ll a_* \sim 1$, the a^{-3} term dominates in (2.4), the Hubble parameter behaves like $H \sim a^{-\frac{3}{2}}$, which means that the universe is in a dust phase. While at late epoch $a \gg a_* \sim 1$, the a^0 term dominates in (2.4), as a result, $H = \text{const}$, which says that the universe enters a de Sitter phase at late time. Here a_* is the transition point between these two phases as mentioned above.

We can solve (2.4) analytically as

$$4\pi G\alpha(t - t_i) = -x^3 + x^3\sqrt{1+x^{-3}} + \frac{3}{2}\ln x + \ln\left(1 + \sqrt{1+x^{-3}}\right),$$

where $x = \frac{a}{a_*}$ and t_i is the initial time when $a(t_i) = 0$. At early time $x \ll 1$, $4\pi G\alpha(t - t_i) = 2x^{\frac{3}{2}}$; while at late time $x \gg 1$, $4\pi G\alpha(t - t_i) = \frac{3}{2}\ln x$. These asymptotic behaviors agree with the previous argument. The numerical relation $t \sim a$ is plotted in Figure 1.

Using (2.4) and the definition of ρ_{DE} , the energy density of DE is

$$\rho_{DE} = \alpha H = \frac{4\pi G}{3}\alpha^2 \left[1 + \sqrt{1 + \left(\frac{a_*}{a}\right)^3}\right].$$

The behavior of ρ_{DE} in terms of a is shown in Figure 2. The equation of state (EoS) of the ghost DE is given by

$$\begin{aligned} \omega &\equiv -\frac{1}{3}\frac{\dot{\rho}_{DE}}{H\rho_{DE}} - 1 = \frac{1}{2}\left(\frac{a_*}{a}\right)^3 \left(\frac{1}{\sqrt{1 + \left(\frac{a_*}{a}\right)^3}} - \frac{1}{1 + \sqrt{1 + \left(\frac{a_*}{a}\right)^3}}\right) - 1 \\ &= \begin{cases} -\frac{1}{2} & a \ll a_* \\ -1 & a \gg a_* \end{cases}. \end{aligned} \quad (2.5)$$

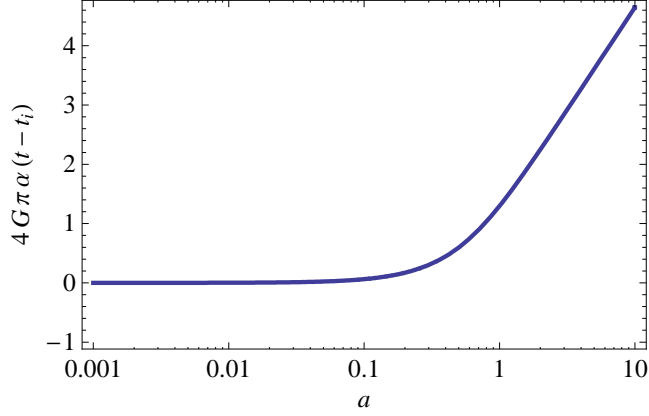


Figure 1: $4\pi G\alpha(t-t_i) \sim a$, where $a_* = 1$

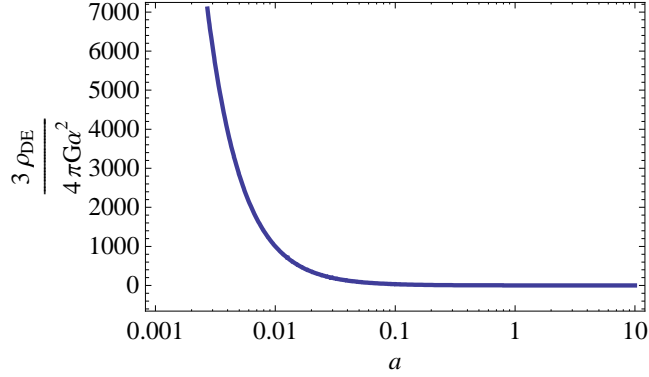


Figure 2: $\frac{3H}{4\pi G\alpha} \sim a$, where $a_* = 1$

From the asymptotic behavior of ω , we can see that the DE acts like a cosmological constant at late time. We plot the relation $\omega \sim a$ in Figure 3. From the figure, we can see that ω can never cross -1 , which is similar to the behavior of quintessence. We will show that this behavior can be altered in the presence of interaction between DE and CDM in section 4. ω varies from $-\frac{1}{2}$ at early time to -1 at late time, which is similar to freezing quintessence model [29]. We can also find that ω has a sharp variation round $a = a_*$. It is easy to understand if we rewrite the expression of ω as

$$3(1 + \omega) = -\frac{\dot{\rho}_{DE}}{H\rho_{DE}} = -\frac{\dot{H}}{H^2} = (H^{-1})', \quad (2.6)$$

from this equation, we can see that in this model the EOS of DE tightly relates to the variation of Hubble parameter, which is quite different in different phases of the universe. For instance, in the dust phase, $(H^{-1})' \sim \frac{3}{2}$; while in the de Sitter phase, $(H^{-1})' \sim 0$. Therefore, there will be a jump from $-\frac{1}{2}$ to -1 when the universe transits from the dust phase to the de Sitter phase.

One can easily show that the EoS of the ghost DE model is $\omega = -\frac{1}{\Omega_m+1}$. The value

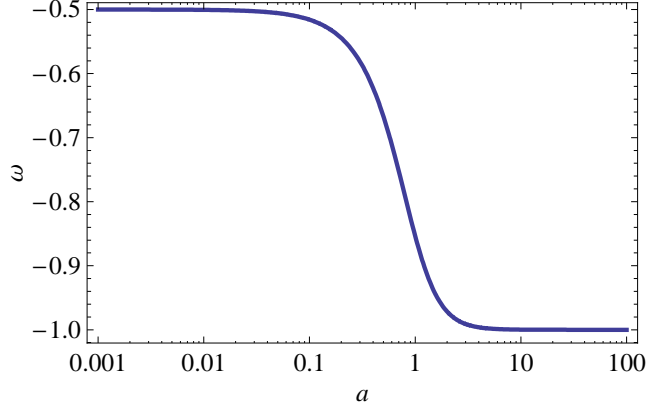


Figure 3: $\omega \sim a$, where $a_* = 1$

ω_0 at present is

$$\omega_0 (a = 1) = \frac{1}{\Omega_{DE0} - 2} = -\frac{1}{\Omega_{m0} + 1}, \quad (2.7)$$

where we have used $a_* = \left(4 \frac{\Omega_{m0}}{\Omega_{DE0}^2}\right)^{\frac{1}{3}}$ in the first equality. This relation is important and helpful to understand the fitting results in section 3.

Also from Raychaudhuri equation (2.3), we can get the total equation of state of the universe

$$\omega_{tot} = -1 - \frac{2}{3} \frac{\dot{H}}{H^2} = 1 + 2\omega = \begin{cases} 0 & a \ll a_* \\ -1 & a \gg a_* \end{cases} \quad (2.8)$$

ω_{tot} decreases monotonically from 0 to -1 , which means that the expansion of the universe switches from deceleration at early epoch to acceleration at late epoch. The conversion occurs at a_{acc} when $\omega_{tot}(a_{acc}) = -\frac{1}{3}$. From (2.5) and (2.8), we can calculate $a_{acc} = \frac{a_*}{2} \sim 0.5$ which means that the universe begin to accelerate at redshift $z_{acc} \sim 1$.

3. Data Fitting

3.1 Model

In order to fit the model with current observational data, we consider a more realistic model which includes DE, CDM, radiation and baryon in a flat FRW universe in this section. In this case, the Friedman equation reads

$$H^2 = \frac{8\pi G}{3} (\alpha H + \rho_{DM} + \rho_b + \rho_r),$$

which can be rewritten as

$$E \equiv \frac{H}{H_0} = \frac{1}{2} \Omega_{DE0} + \sqrt{\frac{1}{4} \Omega_{DE0}^2 + (\Omega_{DM0} + \Omega_{b0}) (1+z)^3 + \Omega_{r0} (1+z)^4},$$

where $\Omega_{DM0}, \Omega_{b0}, \Omega_{r0}$ are present values of dimensionless energy density for CDM, baryon and radiation, respectively. $\Omega_{DE0} = \frac{8\pi G\alpha}{3H_0}$ is dimensionless energy density of DE at present. Energy density of baryon and CDM are always written together as $\Omega_{DM0} + \Omega_{b0} = \Omega_{m0}$. Notice that $\Omega_{DE0} + \Omega_{m0} + \Omega_{r0} = 1$ since we assume a flat universe. The energy density of radiation is the sum of those of photons and relativistic neutrinos

$$\Omega_{r0} = \Omega_{\gamma0} (1 + 0.2271N_n),$$

where $N_n = 3.04$ is the effective number of neutrino species and $\Omega_{\gamma0} = 2.469 \times 10^{-5} h^{-2}$ for $T_{cmb} = 2.725K$ ($h = H_0/100 \text{ Mpc} \cdot \text{km} \cdot \text{s}^{-1}$).

It is worth noticing that from the definition of dimensionless energy density of DE and flatness of our universe we can get an important relation (see also (4.4))

$$(1 - \Omega_{m0}) H_0 = \frac{8\pi G\alpha}{3} = \text{const}, \quad (3.1)$$

where we have neglected Ω_{r0} , which is very small compared to Ω_{m0} . It means that parameters Ω_{m0}, h and α are closely related, α can be expressed in terms of Ω_{m0} and h . We will choose Ω_{m0}, h and Ω_{b0} as free parameters of the model in the following analysis. This relation also infers that there exists a strong degeneracy between Ω_{m0} and h , as shown in subsection 3.3.

3.2 Sets of Observational data

We fit our model by employing some observational data including SnIa, BAO, CMB, Hubble evolution data and BBN.

The data for SnIa are the 557 Uion II sample [22]. χ_{sn}^2 for SnIa is obtained by comparing theoretical distance modulus $\mu_{th}(z) = 5 \log_{10}[(1+z) \int_0^z dx/E(x)] + \mu_0$ ($\mu_0 = 42.384 - 5 \log_{10} h$) with observed μ_{ob} of supernovae:

$$\chi_{sn}^2 = \sum_i^{557} \frac{[\mu_{th}(z_i) - \mu_{ob}(z_i)]^2}{\sigma^2(z_i)}.$$

To reduce the effect of μ_0 , we expand χ_{sn}^2 with respect to μ_0 [30] :

$$\chi_{sn}^2 = A + 2B\mu_0 + C\mu_0^2 \quad (3.2)$$

where

$$\begin{aligned} A &= \sum_i \frac{[\mu_{th}(z_i; \mu_0 = 0) - \mu_{ob}(z_i)]^2}{\sigma^2(z_i)}, \\ B &= \sum_i \frac{\mu_{th}(z_i; \mu_0 = 0) - \mu_{ob}(z_i)}{\sigma^2(z_i)}, \\ C &= \sum_i \frac{1}{\sigma^2(z_i)}. \end{aligned}$$

(3.2) has a minimum as

$$\tilde{\chi}_{sn}^2 = \chi_{sn,min}^2 = A - B^2/C,$$

which is independent of μ_0 . In fact, it is equivalent to performing an uniform marginalization over μ_0 , the difference between $\tilde{\chi}_{sn}^2$ and the marginalized χ_{sn}^2 is just a constant [30]. We will adopt $\tilde{\chi}_{sn}^2$ as the goodness of fit between theoretical model and SnIa data.

The second set of data is the Baryon Acoustic Oscillations (BAO) data from SDSS DR7 [23], the datapoints we use are

$$d_{0.2} = \frac{r_s(z_d)}{D_V(0.2)}$$

and

$$d_{0.35} = \frac{r_s(z_d)}{D_V(0.35)},$$

where $r_s(z_d)$ is the comoving sound horizon at the baryon drag epoch [31], and

$$D_V(z) = \left[\left(\int_0^z \frac{dx}{H(x)} \right)^2 \frac{z}{H(z)} \right]^{1/3}$$

encodes the visual distortion of a spherical object due to the non Euclidianity of a FRW spacetime. The inverse covariance matrix of BAO is

$$C_{M,bao}^{-1} = \begin{pmatrix} 30124 & -17227 \\ -17227 & 86977 \end{pmatrix}.$$

The χ^2 of the BAO data is constructed as:

$$\chi_{bao}^2 = Y^T C_{M,bao}^{-1} Y,$$

where

$$Y = \begin{pmatrix} d_{0.2} - 0.1905 \\ d_{0.35} - 0.1097 \end{pmatrix}.$$

The CMB datapoints we will use are (R, l_a, z_*) from WMAP7 [24]. z_* is the redshift of recombination [32], R is the scaled distance to recombination

$$R = \sqrt{\Omega_m^{(0)}} \int_0^{z_*} \frac{dz}{E(z)},$$

and l_a is the angular scale of the sound horizon at recombination

$$l_a = \pi \frac{r(a_*)}{r_s(a_*)},$$

where $r(z) = \int_0^z dx/H(x)$ is the comoving distance and $r_s(a_*)$ is the comoving sound horizon at recombination

$$r_s(a_*) = \int_0^{a_*} \frac{c_s(a)}{a^2 H(a)} da,$$

where the sound speed $c_s(a) = 1/\sqrt{3(1 + \bar{R}_b a)}$ and $\bar{R}_b = 3\Omega_b^{(0)}/4\Omega_\gamma^{(0)}$ is the photon-baryon energy density ratio.

parameter	Ω_{m0}	h	Ω_{b0}
best-fit _{${}^{+1\sigma, -2\sigma}_{+1\sigma, +2\sigma}$}	$0.257^{+0.009, +0.020}_{-0.016, -0.026}$	$0.662^{+0.011, +0.021}_{-0.011, -0.019}$	$0.054^{+0.001, +0.002}_{-0.002, -0.004}$

Table 1: The best-fit values with 1σ and 2σ errors for Ω_{m0} , h and Ω_{b0} in the ghost dark energy model.

The χ^2 of the CMB data is constructed as:

$$\chi_{cmb}^2 = X^T C_{M,cmb}^{-1} X,$$

where

$$X = \begin{pmatrix} l_a - 302.09 \\ R - 1.725 \\ z_* - 1091.3 \end{pmatrix}$$

and the inverse covariance matrix

$$C_{M,cmb}^{-1} = \begin{pmatrix} 2.305 & 29.698 & -1.333 \\ 29.698 & 6825.270 & -113.180 \\ -1.333 & -113.180 & 3.414 \end{pmatrix}.$$

The fourth set of observational data is 12 Hubble evolution data from [25] and [26]. Its χ_H^2 is defined as

$$\chi_H^2 = \sum_{i=1}^{12} \frac{[H(z_i) - H_{ob}(z_i)]^2}{\sigma_i^2}.$$

Note that the redshift of these data falls in the region $z \in (0, 1.75)$.

The last set we will use is the Big Bang Nucleosynthesis (BBN) data from [27, 28], whose χ^2 is

$$\chi_{bbn}^2 = \frac{(\Omega_{b0} h^2 - 0.022)^2}{0.002^2}.$$

In summary, we have

$$\chi_{tot}^2 = \tilde{\chi}_{sn}^2 + \chi_{cmb}^2 + \chi_{bao}^2 + \chi_H^2 + \chi_{bbn}^2$$

and we assume uniform priors on all the parameters.

3.3 Fitting Results

The best-fit values and errors of parameters are summarized in Table 1. We also list the best-fit values of the corresponding parameters in Table 2 for comparison. The best-fit values of Ω_{m0} and h are slightly smaller than corresponding ones in the Λ CDM model. In Figure 4, we plot the 1D marginalized distribution probability of each parameter. The 2D contour is plotted in Figure 5, from which we can see that there exists a strong correlation between Ω_{m0} and h as we expected in subsection 3.1.

With the best-fit value of $\Omega_{m0} = 0.257$, the transition between the dust phase and the de Sitter phase occurs at $a_* = \left(4 \frac{\Omega_{m0}}{\Omega_{DE}^2}\right)^{\frac{1}{3}} = 1.23$. The universe begins to accelerate

parameter	Ω_{m0}	h	Ω_{b0}
best-fit	0.273	0.703	0.045

Table 2: The best-fit values for the Λ CDM model, using the same data sets.

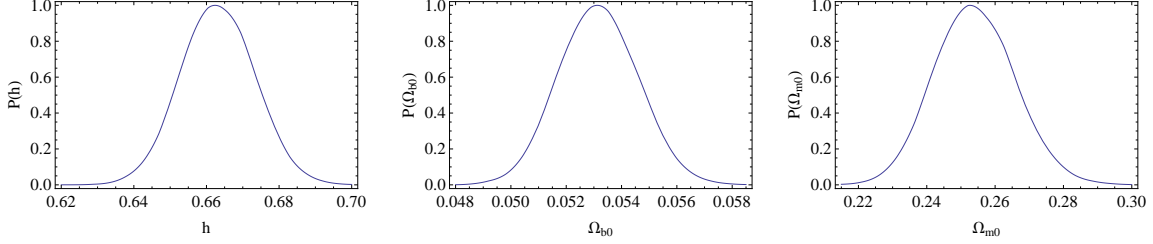


Figure 4: 1D marginalized distribution probability of h , Ω_{b0} and Ω_{m0} .

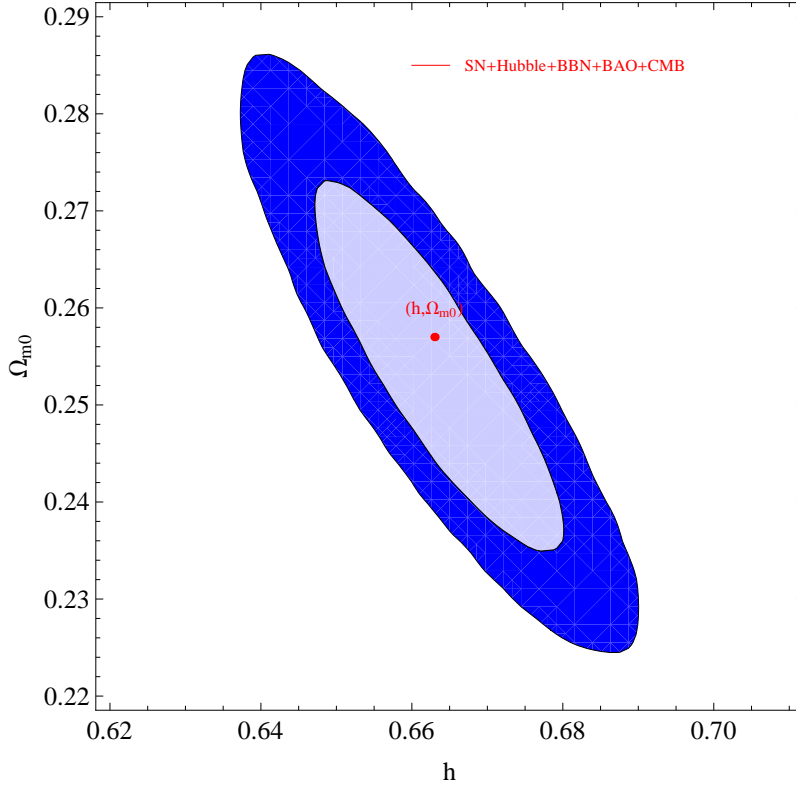


Figure 5: 68% and 95% contour plot in $\Omega_{m0} - h$ plane. The red dot in the figure stands for the best-fit value.

at $a_{acc} = \frac{a_*}{2} = 0.615$, or in terms of redshift, $z_{acc} = 0.625$. And the present EoS of DE $\omega_0 = -\frac{1}{\Omega_{m0}+1} = -0.796$.

χ^2 of best-fit value of this model is $\chi^2_{min} = 607.192$ for $dof = 575$. The reduced χ^2 equals to 1.056 which is acceptable. But χ^2_{min} is larger than the one for the Λ CDM model, $\chi^2_{\Lambda CDM} = 554.264$. A similar conclusion is also reached by other authors using different data set [33].

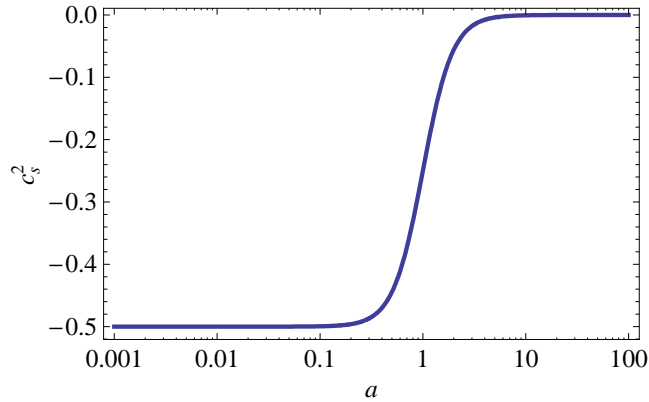


Figure 6: $c_s^2 \sim a$ where $a_* = 1$

It is not hard to understand why in this model, χ_{min}^2 is large, compared to the Λ CDM model. Recently many model independent studies on dynamics of DE show that current observational data favor $\omega \sim -1$, at least at low redshift [34]. But from (2.7) we can see that $\omega_0 \sim -1$ requires a small Ω_{m0} ($\Omega_{m0} \sim 0$), which goes against CMB and BAO observation. As a result, when we combine these different observational data sets to do joint likelihood analysis, the final χ^2 becomes large.

4. Adiabatic Sound Speed and Interaction

In this section, we will return to the simplified model introduced in section 2, and further study this model, where there are only two components, DE and CDM in a flat universe.

The squared adiabatic sound speed of the ghost DE model is found to be

$$c_s^2 = \frac{\dot{p}}{\dot{\rho}_{DE}} = -\frac{1}{2} \frac{1}{\left(\frac{a}{a_*}\right)^3 + 1} < 0.$$

The evolution behavior of the squared adiabatic sound speed is shown in Figure 6. One can see from the figure that c_s^2 leaps from $-\frac{1}{2}$ to 0 at the time a_* . Before a_* , c_s^2 is less than zero, but after $a_* \sim 1$, c_s^2 is approximate to zero. However, it is always negative!

Negative squared adiabatic sound speed may cause some problems. For example, if a fluid evolves adiabatically and has no interaction with gravity or other fluids, negative squared sound speed implies an instability under perturbations. To see this clearly, let us consider a Newtonian argument. Making use of Euler equation, continuity equation of fluid and Poisson equation in Newtonian gravity, we have [38]

$$\ddot{\delta} + 2H\dot{\delta} + c_s^2 k^2 \delta - 4\pi G \bar{\rho} \delta = S_Q + S_{in},$$

where δ is energy density perturbation of the fluid. The terms on the r.h.s are source terms. The first term S_Q represents interaction with other fluid, while the last term S_{in} stands for intrinsic entropy perturbation in the fluid itself. The $4\pi G$ term on the l.h.s represents the

effect of gravity. If there are no source terms, the negative c_s^2 term leads to an instability. But the presence of source terms will change δ 's behavior and makes the thing complicated.

In the ghost DE model, DE interacts with gravity only and there is no S_Q or S_{in} term. Thus the squared sound speed criterion mentioned above shows DE will be unstable under perturbation in Newtonian limit, where we have neglected relativistic effects. However in a full relativistic treatment to discuss the stability of this DE model under linear perturbation, we need to define gauge invariant variables and solve perturbed Einstein's equation and conservation equations. However, it is beyond the scope of this note.

From the above discussion, we see that negative squared adiabatic sound speed may cause a potential instability. But it can be improved if the source terms are present [39]. Therefore in the rest of this section, we introduce the direct interaction between DE and CDM and study the evolution dynamics of the model.

In this case, Friedman equation still reads

$$H^2 = \frac{8\pi G}{3}(\alpha H + \rho_m)$$

and conservation equations are modified to be

$$\begin{aligned}\dot{\rho}_m + 3H\rho_m &= Q, \\ \dot{\rho}_{DE} + 3(1 + \omega)H\rho_{DE} &= -Q,\end{aligned}$$

where Q denotes the interaction between DE and CDM. Since a generic form of Q is not available, we consider three forms which are often discussed in the literature: $Q = 3\bar{\alpha}H\rho_{DE}$, $3\bar{\beta}H\rho_m$ and $3\bar{\gamma}H\rho_{tot}$, where $\bar{\alpha}$, $\bar{\beta}$, and $\bar{\gamma}$ are three constants, (see e.g. [35] for more references). These forms imply that energy transfers in a Hubble time is proportional to energy density of DE, CDM and DE+CDM, respectively, and that energy transfers from DE to CDM if $\bar{\alpha}, \bar{\beta}, \bar{\gamma} > 0$, and vice versa.

In terms of dimensionless quantities, we have

$$1 = \Omega_{DE} + \Omega_m, \tag{4.1}$$

$$\frac{8\pi G}{3H^2}Q = \dot{\Omega}_m + 2\frac{\dot{H}}{H}\Omega_m + 3H\Omega_m, \tag{4.2}$$

$$-\frac{8\pi G}{3H^2}Q = \dot{\Omega}_{DE} + 2\frac{\dot{H}}{H}\Omega_{DE} + 3(1 + \omega)H\Omega_{DE}, \tag{4.3}$$

where $\Omega_{DE} = \frac{8\pi G}{3H^2}\rho_{DE}$, $\Omega_m = \frac{8\pi G}{3H^2}\rho_m$ is dimensionless energy density of DE and CDM, respectively. In addition, by use of the linear relation between ρ_{DE} and H , we have

$$H\Omega_{DE} = H_0\Omega_{DE0} = \text{const.} \tag{4.4}$$

Combining (4.1) and (4.2), one can get

$$-\dot{\Omega}_{DE} + 2\frac{\dot{H}}{H}(1 - \Omega_{DE}) + 3H(1 - \Omega_{DE}) = \frac{8\pi G}{3H^2}Q, \tag{4.5}$$

Putting (4.3) to eliminate Q , we arrive at

$$2\dot{H} + 3\omega H^2\Omega_{DE} + 3H^2 = 0. \tag{4.6}$$

Then from (4.5) and (4.4), one can have the equation of motion of Ω_{DE} as

$$-\dot{\Omega}_{DE} \frac{2 - \Omega_{DE}}{\Omega_{DE}} + 3H_0 \Omega_{DE0} \frac{1 - \Omega_{DE}}{\Omega_{DE}} = \frac{8\pi G}{3H^2} Q \equiv H_0 \Omega_{DE0} \Omega_Q, \quad (4.7)$$

where

$$\Omega_Q = \begin{cases} 3\bar{\alpha}, & \text{when } Q = 3\bar{\alpha}H\rho_{DE} \\ 3\bar{\beta} \frac{1 - \Omega_{DE}}{\Omega_{DE}}, & \text{when } Q = 3\bar{\beta}H\rho_m \\ 3\bar{\gamma} \frac{1}{\Omega_{DE}}, & \text{when } Q = 3\bar{\gamma}H\rho_{tot}. \end{cases}$$

Expressing this equation in terms of efolding-number $N \equiv \ln a$, and making use of $\frac{d\Omega_{DE}}{dt} = \frac{d\Omega_{DE}}{dN} H_0 \Omega_{DE0} / \Omega_{DE}$, we obtain

$$-\Omega'_{DE} \frac{2 - \Omega_{DE}}{\Omega_{DE}^2} + 3 \frac{1 - \Omega_{DE}}{\Omega_{DE}} = \Omega_Q. \quad (4.8)$$

Using (4.4), (4.6) and (4.7), we can get the EoS of the DE

$$\omega = -\frac{1}{2 - \Omega_{DE}} - \frac{2}{3} \frac{\Omega_Q}{2 - \Omega_{DE}}, \quad (4.9)$$

while the EoS of the total fluid is

$$\omega_{tot} = -1 - \frac{2}{3} \frac{\dot{H}}{H^2} = \omega \Omega_{DE}, \quad (4.10)$$

in the second equality, we have used (4.6). In addition, the deceleration parameter is given by

$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1 - 2\Omega_{DE}}{2 - \Omega_{DE}} - \frac{\Omega_Q \Omega_{DE}}{2 - \Omega_{DE}}, \quad (4.11)$$

and the squared speed of sound reads

$$c_s^2 = \frac{\dot{p}}{\dot{\rho}_{DE}} = \left(\Omega_{DE} \frac{d}{d\Omega_{DE}} - 1 \right) \frac{1 + \frac{2}{3}\Omega_Q}{2 - \Omega_{DE}}. \quad (4.12)$$

We will solve Ω_{DE} , ω , ω_{tot} and c_s^2 analytically for each Q in the following. However, for the sake of brevity, we will discuss the case of $Q = 3\bar{\alpha}H\rho_{DE}$ only in some detail.

4.1 $Q = 3\bar{\alpha}H\rho_{DE}$

In this case, (4.8) becomes

$$-\Omega'_{DE} (2 - \Omega_{DE}) + 3\Omega_{DE} - 3(\bar{\alpha} + 1)\Omega_{DE}^2 = 0.$$

Its solution is

$$3N + C = 2 \ln \Omega_{DE} - \frac{1 + 2\bar{\alpha}}{1 + \bar{\alpha}} \ln |1 - (\bar{\alpha} + 1)\Omega_{DE}|, \quad (4.13)$$

where the integration constant $C = 2 \ln \Omega_{DE0} - \frac{1 + 2\bar{\alpha}}{1 + \bar{\alpha}} \ln |1 - (\bar{\alpha} + 1)\Omega_{DE0}|$, Ω_{DE0} is the dimensionless energy density of DE at present.

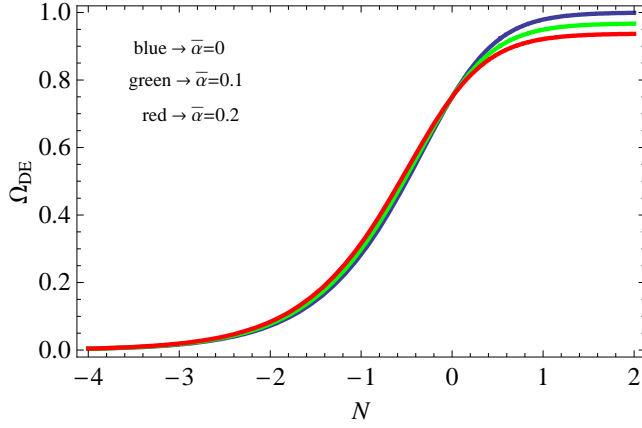


Figure 7: $\Omega_{DE} \sim N$ where N is e-folding-number. Here $\Omega_{DE0} = 0.75$. The blue, green and red curves correspond to $\bar{\alpha} = 0, 0.1$ and 0.2 , respectively.

If $\bar{\alpha} < 0$, from the solution (4.13), we can see that Ω_{DE} can be larger than 1 at late time which is unphysical. This unphysical result comes from the fact that the assumption on energy transfer rate $Q \propto \rho_{DE}$ is oversimplified. For $\bar{\alpha} < 0$, DE will gain energy from CDM and energy density of CDM will become less and less. At some time, ρ_m becomes zero, and it is impossible to keep on transferring energy to DE. But due to the oversimplified assumption on Q , CDM will continue to lose energy which is incorrect physically. Therefore, we will presume $\alpha > 0$ in this subsection.

From (4.13) one has $(\bar{\alpha} + 1)\Omega_{DE} < 1$. It means that for $\bar{\alpha} > 0$, Ω_{DE} will tend to a constant $1/(1 + \bar{\alpha})$, rather than 1. The relation of $\Omega_{DE} \sim N$ is shown in Figure 7, it also indicates that when $\bar{\alpha}$ is larger, the evolution of Ω_{DE} will be flatter since more energy are injected into CDM. Of course there is a upper limit for $\bar{\alpha}$, as Ω_{DE} must be able to reach its present value $\Omega_{DE0} \sim 0.75$. For $\bar{\alpha} > 0$, the coincidence problem can be alleviated excellently, and if $\bar{\alpha} \sim -1 + 1/\Omega_{DE0}$, this problem is completely solved. The similar situation occurs as $Q \sim \rho_{tot}$ (see section 4.3).

In this case, the equation of state of DE is

$$\omega = -\frac{1 + 2\bar{\alpha}}{2 - \Omega_{DE}}.$$

We plot the relation $\omega \sim N$ in Figure 8. We have shown in section 2, it is impossible for ω to cross phantom divide without interaction. Nevertheless from this figure we can see that the situation is changed with the help of interaction term. ω will cross -1 from the quintessence regime to phantom regime and approach to $-1 - 2\bar{\alpha}$ at late time when $\bar{\alpha} \neq 0$.

The equation of state of the total fluid is

$$\omega_{tot} = -(1 + 2\bar{\alpha}) \frac{\Omega_{DE}}{2 - \Omega_{DE}}$$

and it is plotted in Figure 9. As we expect again, ω_{tot} can be smaller than -1 in the presence of interaction term and reaches its asymptotic value $-1 - 2\bar{\alpha} < -1$ at late time. Therefore, in this model the universe will end with the big rip singularity in the future [36, 37]. And

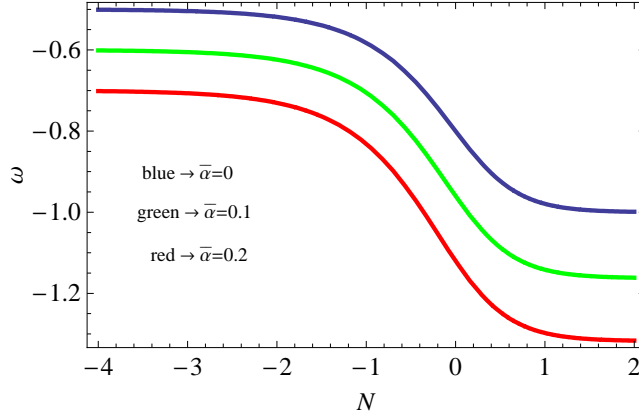


Figure 8: $\omega \sim N$ where N is e-folding-number. Here $\Omega_{DE0} = 0.75$. The blue, green and red curves correspond to $\bar{\alpha} = 0, 0.1$ and 0.2 , respectively.

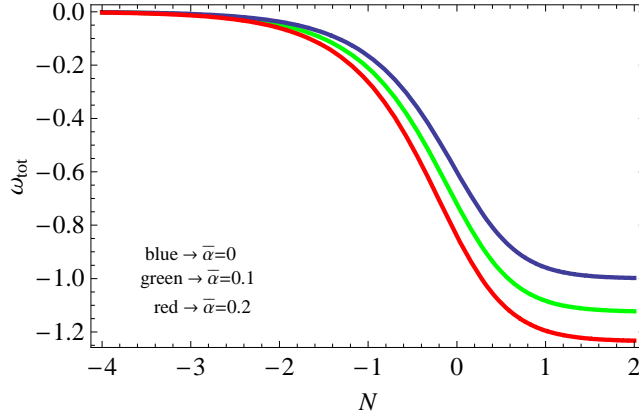


Figure 9: $\omega_{tot} \sim N$ where N is e-folding-number. Here $\Omega_{DE0} = 0.75$. The blue, green and red curves correspond to $\bar{\alpha} = 0, 0.1$ and 0.2 , respectively.

from this figure, we can see that the universe begins to accelerate earlier with larger $\bar{\alpha}$. Note that according to the calculations in section 2 and section 3, the universe begins to accelerate at $z_{acc, \bar{\alpha}=0} = 0.6$ in the case without interaction. Thus the acceleration of the universe occurs at $z_{acc, \bar{\alpha}} > 0.6$ when the interaction is present if one keeps $\Omega_{m0} = 0.257$.

Finally, the squared speed of sound reads

$$c_s^2 = \frac{\dot{p}}{\dot{\rho}_{DE}} = -2(1 + 2\bar{\alpha}) \frac{1 - \Omega_{DE}}{(2 - \Omega_{DE})^2}, \quad (4.14)$$

the squared sound speed is always smaller than 0 because we assumed $\bar{\alpha} > 0$. This means that this form of interaction cannot make c_s^2 positive.

4.2 $Q = 3\bar{\beta}H\rho_m$

In this case, (4.8) becomes

$$-(2 - \Omega_{DE})\Omega'_{DE} + 3(1 - \bar{\beta})\Omega_{DE}(1 - \Omega_{DE}) = 0.$$

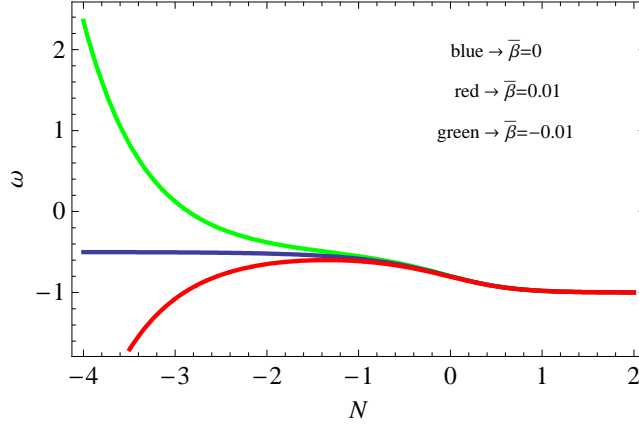


Figure 10: $\omega \sim N$ where N is e-folding-number. Here $\Omega_{DE0} = 0.75$. Blue, green and red curves correspond to $\bar{\beta} = 0, -0.01$ and 0.01 , respectively.

its analytical solution reads

$$3(1 - \beta)N + C = 2 \ln \Omega_{DE} - \ln(1 - \Omega_{DE}),$$

where the integration constant $C = 2 \ln \Omega_{DE0} - \ln(1 - \Omega_{DE0})$. We can see from the analytical solution that Ω_{DE} varies from 0 at early time to 1 at late time. The EoS of DE is

$$\omega = -\frac{1}{2 - \Omega_{DE}} - 2\bar{\beta} \frac{1}{2 - \Omega_{DE}} \frac{1 - \Omega_{DE}}{\Omega_{DE}}.$$

One can see in this case that at early time $\omega \rightarrow -\bar{\beta}/\Omega_{DE}$ as $\Omega_{DE} \rightarrow 0$. The plot is shown in Figure 10. If $\bar{\beta} > 0$, ω will increase from $-\infty$ to some local maximum at some point and then decrease to its asymptotic value -1 at late time. Unlike the situation we have discussed in subsection 4.1, ω will cross -1 from phantom regime to quintessence regime in this case. If $\bar{\beta} < 0$, ω will decay monotonically from $+\infty$ to -1 and never cross -1 . However, no matter what the value of $\bar{\beta}$ is, the late-time asymptotic behaviors of ω are all the same.

The EoS of the total fluid is

$$\omega_{tot} = -\frac{2\bar{\beta} + (1 - 2\bar{\beta})\Omega_{DE}}{2 - \Omega_{DE}} = \begin{cases} -\bar{\beta} & \text{at early time} \\ -1 & \text{at late time.} \end{cases}$$

The reasonable value of $\bar{\beta}$ should be $|\bar{\beta}| < 1$. Thus the big rip singularity will be avoided in this case.

Finally, we give the squared speed of sound

$$c_s^2 = -2 \frac{1 - \Omega_{DE}}{(2 - \Omega_{DE})^2} - 2\bar{\beta} \left[\frac{4 - 5\Omega_{DE} + 2\Omega_{DE}^2}{(2 - \Omega_{DE})^2 \Omega_{DE}} \right],$$

from which we can find that $c_s^2 \rightarrow -2\bar{\beta}/\Omega_{DE}$ at early time. Therefore, this interaction term will not make c_s^2 positive as well if $\bar{\beta} > 0$. Furthermore, at early time the speed of sound will be larger than speed of light if $\bar{\beta} < 0$.

4.3 $Q = 3\bar{\gamma}H\rho_{tot}$

In this case (4.8) becomes

$$-\Omega'_{DE} \frac{2 - \Omega_{DE}}{\Omega_{DE}^2} + 3 \frac{1 - \Omega_{DE}}{\Omega_{DE}} = 3\bar{\gamma} \frac{1}{\Omega_{DE}}.$$

Its analytical solution reads

$$3N + C = \frac{2}{1 - \bar{\gamma}} \ln \Omega_{DE} - \left(\frac{1 + \bar{\gamma}}{1 - \bar{\gamma}} \right) \ln |1 - \bar{\gamma} - \Omega_{DE}|,$$

where $C = \frac{2}{1 - \bar{\gamma}} \ln \Omega_{DE0} - \left(\frac{1 + \bar{\gamma}}{1 - \bar{\gamma}} \right) \ln |1 - \bar{\gamma} - \Omega_{DE0}|$. The EoS of DE

$$\omega = -\frac{1}{2 - \Omega_{DE}} - 2 \frac{\bar{\gamma}}{2 - \Omega_{DE}} \frac{1}{\Omega_{DE}}.$$

Once again ω will diverge at early time when the interaction is present. The EoS of the total fluid is

$$\omega_{tot} = -\frac{2\bar{\gamma}}{2 - \Omega_{DE}} - \frac{\Omega_{DE}}{2 - \Omega_{DE}},$$

and the squared speed of sound reads

$$c_s^2 = -2 \frac{1 - \Omega_{DE}}{(2 - \Omega_{DE})^2} + 2\bar{\gamma} \frac{3\Omega_{DE} - 4}{\Omega_{DE} (2 - \Omega_{DE})^2}$$

which is also divergent at early time $\Omega_{DE} \rightarrow 0$.

5. Conclusion and Discussion

In this note we investigated a DE model whose energy density is proportional to Hubble parameter with a coefficient which is roughly order of Λ_{QCD}^3 . It gives the right order of magnitude of observed energy density of DE. We studied its cosmological evolution. In this DE model, the universe has a de Sitter phase at late time and begins to accelerate at redshift around $z_{acc} \sim 0.6$.

We also fitted this model with observational data including SnIa, BAO, CMB, BBN and Hubble parameter data. The best-fit values of parameters of the model are $\Omega_{m0} = 0.257$, $h = 0.662$, $\Omega_{b0} = 0.054$. However, the minimal χ^2 gives $\chi_{min}^2 = 607.192$, while in the Λ CDM model, $\chi_{\Lambda CDM}^2 = 554.264$ for the same data sets. Namely the simple χ^2 analysis seemingly implies that current data do not favor the ghost DE model, compared to the Λ CDM model. Clearly this result is not conclusive, further study is needed.

We also found that the squared sound speed of the DE is negative, which may give rise to a potential instability of the model under perturbation. We further studied the cosmological dynamics of the model by considering there exists some interaction between DE and CDM. Three kinds of interaction forms are discussed. In all cases, the negative squared sound speed is still there with a proper coefficient for the interaction terms. Clearly the potential instability should be studied seriously by investigating linearized Einstein equations, not just calculating the squared sound speed of the DE, which is currently under investigation. If the instability indeed exists, then the ghost DE model has to be further modified or to be abandoned.

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